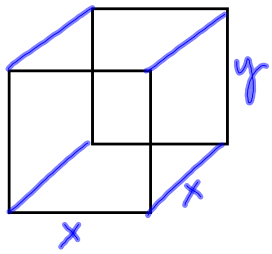
Optimization

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?



1) max volume

$f(x,y) = x^2 y$

2) Restriction is S.A = 108

$108 = x^2 + 4xy$

Solve for y

$108 - x^2 = 4xy$

$\frac{108 - x^2}{4x} = y \longrightarrow$ Plug into $f(x,y)$

$f(x) = x^2\left(\frac{108-x^2}{4x}\right) \rightarrow$ function to maximize

$f(x) = x\left(\frac{108-x^2}{4}\right)$ ☆ simplify 1st

$f(x) = \frac{108x - x^3}{4}$

$f(x) = \frac{1}{4}(108x - x^3)$

$f'(x) = \frac{1}{4}(108 - 3x^2)$ Take derivative

$0 = \frac{1}{4}(108 - 3x^2)$

$0 = 108 - 3x^2$ mult by 4

$3x^2 = 108$ add $3x^2$

$x^2 = 36$ $\div 3$

$x = 6 \longrightarrow$ max or min?

$\cancel{x = -6}$

$f''(x) = \frac{1}{4}(-6x)$

$f''(6) = -$

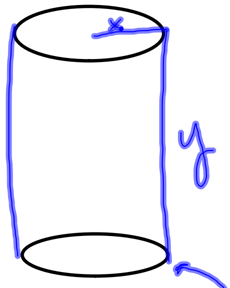
Answer $6 \times 6 \times 3$

May 16-7:16 AM

EXAMPLE 4 Design a cylindrical can of volume 10 ft^3 so that it uses the least amount of metal (Figure 6). In other words, minimize the surface area of the can (including its top and bottom).

minimize : Surface Area

$f(x,y) = 2\pi x^2 + 2\pi xy$



$V = \pi x^2 y$

Restriction: $V = 10$

$10 = \pi x^2 y$

$y = \frac{10}{\pi x^2}$ (Solve for y)

$2\pi x^2 + 2\pi x\left(\frac{10}{\pi x^2}\right) = f(x)$

$f(x) = 2\pi x^2 + \frac{20}{x}$ (Simplify)

$20x^{-1}$

$-20x^{-2}$

$f'(x) = 4\pi x - \frac{20}{x^2} = 0$

$f'(x) = \quad 4\pi x = \frac{20}{x^2}$

$4\pi x^3 = 20$ Cross X

$x^3 = \frac{20}{4\pi}$

$x = \sqrt[3]{\frac{5}{\pi}}$

$-$ $+$

$\sqrt[3]{\frac{5}{\pi}}$

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Which points on the graph of $y = 4 - x^2$ are closest to the point (0, 2)?

$$\lim_{x\to\infty} xe^x = \infty$$

$$f(x) = xe^x \qquad \lim_{x\to-\infty} xe^x = 0$$

$$\frac{-1000000}{e^{1000000}}$$

$$f'(x) = xe^x + e^x$$

$$= e^x(x+1) = 0$$

$$\underset{-1}{-\quad|\quad+}$$

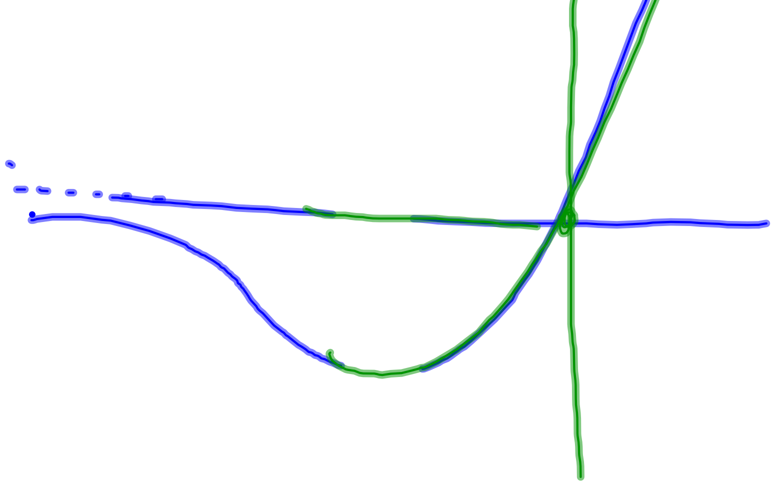
$$f''(x) = e^x + e^x(x+1)$$

$$e^x(1 + x+1)$$

$$e^x(2+x)$$

$$\underset{-2}{-\quad|\quad+}$$

$$\underset{-2 \qquad -1}{-- \quad | \quad -+ \quad | \quad ++}$$



-4π $\quad \lim_{x \to \infty} f(x)$

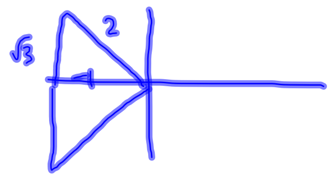
3.

$$y = 2x + 4\sin x \qquad [-2\pi, 2\pi]$$

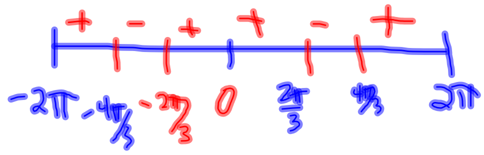
$$y' = 2 + 4\cos x = 0$$

$$4\cos x = -2$$

$$\cos x = -\frac{1}{2}$$

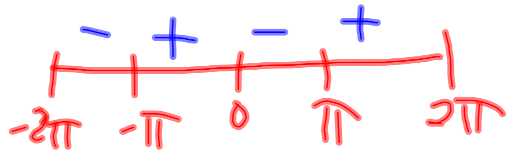


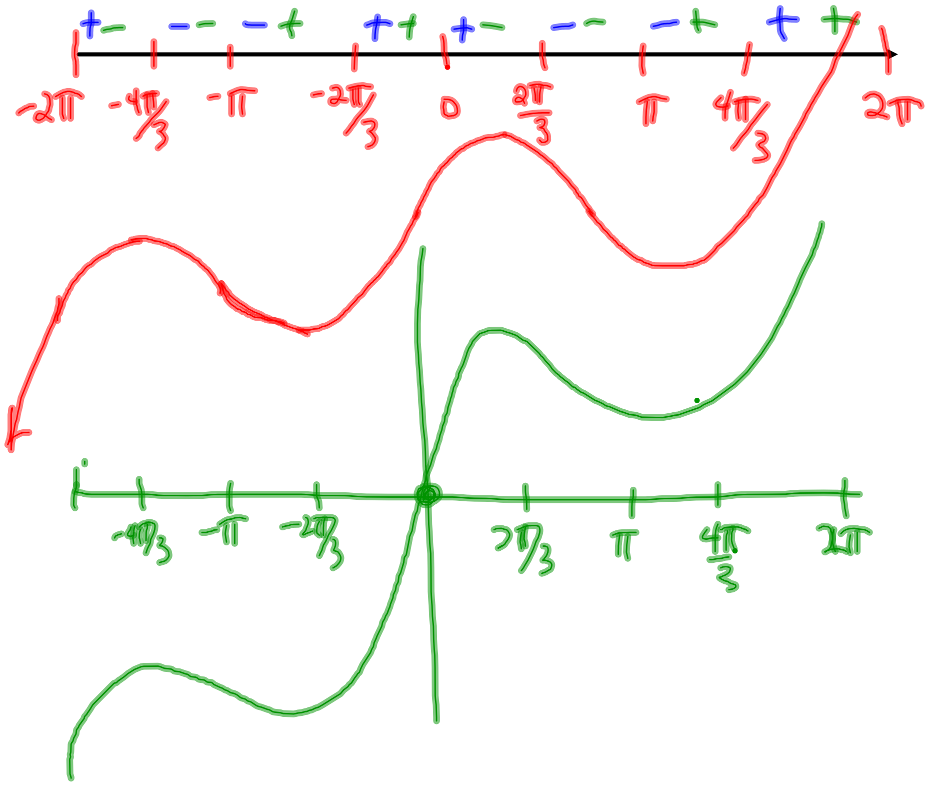
$-\frac{2\pi}{3}, -\frac{4\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$



$$y'' = -4\sin x = 0$$

$-2\pi, -\pi, 0, \pi, 2\pi$

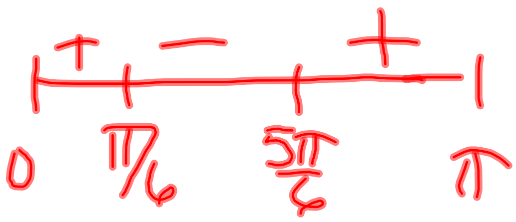




$f(x) = \cos x + \frac{1}{2}x$ $\quad [0, \pi]$

$f'(x) = -\sin x + \frac{1}{2}$

$\sin x = +\frac{1}{2}$



$f'' = -\cos x$